

Experimental Measurement of Lower and Upper Bounds of Concurrence for Mixed Quantum States

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We experimentally measure the lower and upper bounds of concurrence for a set of two-qubit mixed quantum states using photonic systems. The measured concurrence bounds are in agreement with the results evaluated from the density matrices reconstructed through quantum state tomography. In our experiment, we propose and demonstrate a simple method to provide two faithful copies of a two-photon mixed state required for parity measurements: Two photon pairs generated by two neighboring pump laser pulses through optical parametric down conversion processes represent two identical copies. This method can be conveniently generalized for entanglement estimation of multi-photon mixed states.

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Quantum entanglement plays a key role in not only fundamental quantum physics but also quantum information processing. Consequently the characterization and quantification of entanglement have attracted much attention [1, 2, 3, 4, 5, 6] and various entanglement measures have been proposed, such as concurrence [2, 7, 8], negativity [4] and tangle [5]. With these theoretical progresses, experimental quantification of entanglement becomes a natural requirement. However, it is a rather difficult task, since many entanglement measures proposed are complicated nonlinear functions of the density matrix of the quantum state. One simple method to estimate entanglement degree is quantum state tomography [9, 10], which has been applied successfully in a number of experiments [11, 12, 13]. In quantum state tomography, one measures a complete set of observables and reconstructs the density matrix of the measured quantum state, and then the left thing is to mathematically evaluate some entanglement measure using the density matrix. Since one has to measure a complete set of observables for tomography, this leads to rapidly growing experimental overhead as system size increases, either in subsystem dimensions or in subsystem numbers. That makes quantum state tomography an unscalable method. Another important method for entanglement detection is the entanglement witness [14, 15, 16, 17], which provides a much more direct experimental insights to the entanglement property of a quantum state. However, it requires some a priori knowledge on the state to be detected. So entanglement witnesses can not be freely applied for arbitrary unknown quantum states.

To overcome the above drawbacks, Mintert *et al.* [18] recently proposed a method to directly measure the concurrence of an arbitrary pure state $|\Psi\rangle$ through a single

projection measurement on its twofold copy $|\Psi\rangle \otimes |\Psi\rangle$. Based on this method, experimental measurements of concurrence for two-qubit [19, 20] and 4×4 -dimensional [21] pure states have been reported. However, for more general applications one would like to have the ability to experimentally measure entanglement of not only pure states, but also mixed states. For this purpose, Mintert *et al.* [22] and Aolita *et al.* [23] presented observable lower bounds of concurrence for arbitrary bipartite and multipartite mixed states, respectively. After that, some of us [24] presented observable upper bound of concurrence for arbitrary finite-dimensional mixed states.

In this paper, we report the first experimental measurements of lower and upper bounds of concurrence for a set of two-qubit mixed states using the twofold copy parity measurements in [22] and [24]. Our results give an exact region which must contain the concurrence of the measured mixed states. We also reconstruct the density matrices of the mixed states through quantum state tomography and evaluate lower and upper bounds of concurrence with the density matrices. We find that the experiment results obtained by these two methods are in agreement with each other. So far most experiments investigating entanglement properties have invoked photonic systems [11, 12, 13, 19, 21, 25] due to their mature manipulation technologies and wide applications in quantum information science. However an important experimental difficulty to realize the measurements in [22] and [24] using photon systems is the preparation of two identical copies of an unknown mixed state ρ , that is, one who wants to implement these measurements has to be certain of the source providing $\rho \otimes \rho$, but can be perfectly ignorant about the initialization of the source, as pointed out by Mintert *et al.* in [22]. In our experiment, we present and demonstrate a method by which one can easily prepare a reliable photon source providing two faithful copies of an unknown mixed state.

Our method utilizes the copies carried by photons

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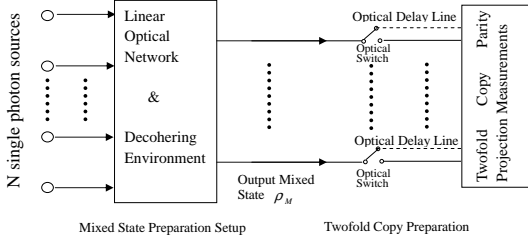


FIG. 1: Scheme for preparing two faithful copies of an unknown multi-photon mixed state.

emitted out from the same photon sources and passing through the same preparation setup, but at different times. The method is depicted in Fig. 1. Suppose in the most general situation of linear optics quantum information processing in the future, we have N single photon sources which can emit one single photon pulse at one time. After passing through some linear optical quantum computing networks and at the same time suffering from some decoherence processes, the output M photons will be in a multi-partite mixed state ρ_M . Now if we want to detect the entanglement of this multi-photon mixed state, we can send each photon into an optical delay line (This can be achieved by sending photons into new optical paths and controlling the length of these new paths as delay line). Then let's wait for the multi-photon state generated by next pulses of these single photon sources. Since all photons pass through exactly the same state preparation setup, this multi-photon state must be in the same mixed state ρ_M . Thus we have got two faithful copies of the same multi-photon mixed state. Moreover, we even don't need to control the state preparation setup. That means this method can provide us two identical copies of any unknown multi-partite mixed state. At last we can send the photons in the new paths and original paths into the parity measurements setup for concurrence bounds measurements as proposed in [23].

Now let us briefly introduce the measurements in [22] and [24]. The lower bound of concurrence for an bipartite mixed state presented in [22] is described by the following inequality

$$[C(\rho)]^2 \geq \text{Tr}(\rho \otimes \rho V_i) \quad (i = 1, 2). \quad (1)$$

Here $V_1 = 4(P_- - P_+) \otimes P_-$ and $V_2 = 4P_- \otimes (P_- - P_+)$. P_- is the projector on the antisymmetric subspace of the two copies of either subsystem and P_+ is the symmetric counterpart of P_- . The upper bound derived in [24] also corresponds to the inequality

$$[C(\rho)]^2 \leq \text{Tr}(\rho \otimes \rho K_i) \quad (i = 1, 2), \quad (2)$$

where $K_1 = 4(P_- + P_+) \otimes P_-$, $K_2 = 4P_- \otimes (P_- + P_+)$. However, for the case of two-qubit mixed states, there is a tighter upper bound [24]

$$[C(\rho)]^2 \leq \text{Tr}(\rho \otimes \rho \cdot 4P_- \otimes P_-) \quad (3)$$

Thus in experiment we only need to make a few parity projection measurements $P_- \otimes P_-$, $P_- \otimes P_+$ and $P_+ \otimes P_-$ on the twofold copy to evaluate the lower and upper bounds.

Our experiment setup is outlined in Fig. 2. Instead of using single photon sources, here we use photon pairs produced by optical parametric down-conversion processes to prepare two-qubit mixed states. A pulse train from a mode-locked Ti:sapphire laser (with a duration of 140 fs, a repetition rate of 76 MHz and a central wavelength of 780 nm) first passes through a frequency doubler. Then the ultraviolet pulses (in H polarization, with 200 mW average power) from the doubler pump two 1 mm thick β -barium borate crystals (BBO1 and BBO2) located side by side to generate polarization-entangled photon pairs. The performance and detailed description of this photon pair source can be found in [26]. With this source, the output photon pairs from single-mode fibers (A and B) are in the maximally entangled state $\frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle)$. Then one photon of the pair (from single-mode fiber A) passes through a phase-damping channel which is composed of a birefringent crystal (we use quartz crystal here). After that the twin-photon is prepared in a certain two-qubit mixed state. Thus by changing the thickness of the quartz crystal we can prepare a set of two-qubit mixed state with different concurrence values. Next step is to prepare a twofold copy of this mixed state. For experiment convenience, we use two 50/50 beamsplitters (BS1 and BS2) in our experiment instead of the optical switches in Fig. 1. The only drawback of this change is the decrease of total detection efficiency. At last another two 50/50 beamsplitters (BS3 and BS4) are used to make the P_- projection measurement on the twofold copy of each subsystem. So the coincidence counts of single photon detectors D1, D2, D3, and D4 correspond to the result of $P_- \otimes P_-$ measurement. To assure BS3 (BS4) making P_- measurement, the optical path lengths between BS1 (BS2) and BS3 (BS4) are carefully arranged so that the reflected photon on BS1 (BS2) can arrive at BS3 (BS4) at the same time with the transmitted photon generated by the next pump laser pulse. In the experiment the reflection arm is about 3.947 meters longer than the transmission arm. This length corresponds to the spatial distance between two neighbouring pump laser pulses, which is $c * \frac{1}{76 \text{ MHz}}$, with c denoting the speed of light. If the two photons have the same polarization, by moving the right angle prism P1 (P2) mounted on a translational stage and observing the two-photon coincidence counts of D1 and D2 (D3 and D4), we can observe a two-photon interference dip, with theoretical visibility 1/3. The interference of these two photons is intrinsically the same as the interference between two photons from two spatially separated photon pair sources [27]. In our experiment, the visibilities of two interferences at BS3 and BS4 are both 0.30. Now if we make the two

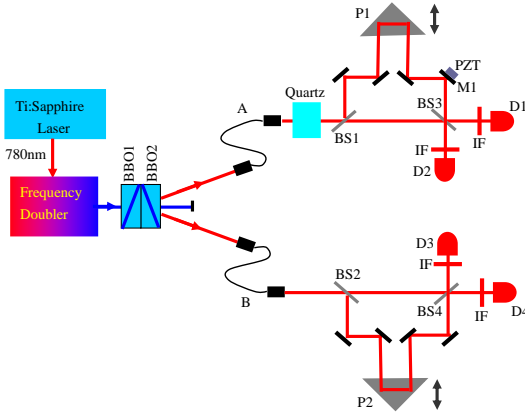


FIG. 2: (Color online) Experiment setup for measuring lower and upper bounds of concurrence for two-photon mixed states. IF is interference filter centered at 780 nm wavelength with 3 nm bandwidth.

translational stages stay exactly at the dip places, the four-photon coincidences should correspond to the result of $P_- \otimes P_-$ measurement.

According to the original scheme in [22], we should also perform $P_- \otimes P_+$ and $P_+ \otimes P_-$ measurements. Here we use a little trick for experimental convenience. Notice that $P_- + P_+ = I$, so we can perform $P_- \otimes I$, $I \otimes P_-$ measurements to evaluate $P_- \otimes P_+$ and $P_+ \otimes P_-$. We also need to perform $I \otimes I$ measurement for normalization. The I measurement on the twofold copy of either subsystem in our experiment setup can be easily realized by moving the translational stage out of the two-photon interference region. In this case, the measured two-photon coincidences of D1 and D2 (D3 and D4) should correspond to $\frac{1}{2}I$ measurement. In this way we can get the results of $\frac{1}{2}I \otimes P_-$, $P_- \otimes \frac{1}{2}I$ and $\frac{1}{2}I \otimes \frac{1}{2}I$ measurements. Combining the measured result of $P_- \otimes P_-$, we can evaluate the lower and upper bounds of concurrence.

In the above analysis, we only consider the case that the first and second pump laser pulses each generates one photon pair. But due to the specialty of spontaneous parametric down conversion source, there is equal probability that two photon pairs are produced by only one pump pulse (the first or the second). These two additional cases also have contributions to four-photon coincidences. So we have to subtract these backgrounds from the above measurement results. To do this, we block the reflection arm of one subsystem and record the four-photon coincidences as $b1$. Similarly, the four-photon coincidences when blocking the transmitted arm of one subsystem are recorded as $b2$. Then the background coincidences are $b1 + b2$. Notice that these backgrounds keep constant against the location of translational stages because the contributions from the additional two cases have no relationship with two-photon interferences on BS3 and BS4. So we can subtract $b1 + b2$ from each measured coincidence counts and finally obtain the net

TABLE I: Experiment results for eight mixed states via two methods. $C_l^{twofold}$ and $C_u^{twofold}$ are lower and upper bounds of concurrence obtained by twofold copy parity projection measurements, respectively C^{tom} , C_l^{tom} , and C_u^{tom} are concurrence and its lower and upper bounds evaluated by tomography, respectively. T_{quartz} is thickness of the quartz crystal for corresponding mixed state.

T_{quartz} (mm)	C_l^{tom}	$C_l^{twofold}$	C^{tom}	$C_u^{twofold}$	C_u^{tom}
0	0.931	0.860 ± 0.063	0.932	0.949 ± 0.027	0.965
2.985	0.908	0.801 ± 0.086	0.910	0.869 ± 0.035	0.955
6.584	0.812	0.611 ± 0.071	0.815	0.812 ± 0.024	0.910
9.568	0.669	0.705 ± 0.084	0.672	0.877 ± 0.031	0.851
13.167	0.539	0.388 ± 0.142	0.539	0.833 ± 0.029	0.803
17.468	0.349	0.297 ± 0.158	0.376	0.686 ± 0.029	0.747
20.453	0.237	0.250 ± 0.213	0.239	0.835 ± 0.029	0.727
24.052	0.00	0.182 ± 0.234	0.092	0.782 ± 0.024	0.703

results of $P_- \otimes P_-$, $\frac{1}{2}I \otimes P_-$, $P_- \otimes \frac{1}{2}I$ and $\frac{1}{2}I \otimes \frac{1}{2}I$.

Another problem should be mentioned is the interference effect between the above three cases. Due to the mode-locked property of the pump laser, the phase of one pump laser pulse is locked with the next pulse. Thus the four-photon coincidences contributions from these three cases should be coherent in phase. Such phase coherence would cause interference effect on four-photon coincidences and could spoil experiment results. To remove this effect, we mount one reflection mirror (M1) on a piezoelectric transducer (PZT) and drive the PZT with a random voltage. This will induce a random phase between each of the three cases and thus destroy the phase coherence of them [28]. In our experiment, due to a very long time required for observing four-photon coincidences, we observe the two-photon coincidences of D1 and D3 instead to make sure that phase coherence has been destroyed. The two-photon coincidences may come from two cases: the first pump pulse generates one photon pair and the photon pair passes through the reflection arm; or the second pulse generates one pair while the photon pair passes through the transmitted arm. These two cases have similar phase coherence with the above three cases. By observing coincidences of D1 and D3, we find that the coincidences varies obviously in the time scale of a few seconds when no voltage is applied on the PZT. This is because the relative phase between the reflection and transmission arm is not stable, as in normal Mach-Zehnder interferometers. But when we drive the PZT with an random signal, the coincidences become stable and no interference effect can be observed. This phenomenon demonstrates that phase coherence between such cases can be effectively destroyed using this method.

The experiment results are listed in Table I. We measured the concurrence bounds of eight mixed states. Eight quartz crystals with different thicknesses ranging from 0 to 24 mm are employed as decohering environment to prepare these mixed states. From Table I we can see that most lower and upper bounds calculated from parity projection measurements are compatible with the

results evaluated through tomography method, considering the error bars caused by photon counting statistics. Furthermore, for most states the concurrence evaluated by tomography are in the region between the lower and upper bounds obtained from parity projection measurements. Here we didn't calculate the density matrices of these mixed states from the parameters of our experiment setup, since state preparation is not the purpose of this experiment. Instead we compare the results of parity projection measurements with the results via tomography, because quantum state tomography has been applied rather successfully for two-qubit cases [9].

Comparing these two methods in such two-qubit case, it seems that the method of twofold copy parity measurements is more complicated and needs more data collection time, since two two-photon interferometers are used and four-photon coincidence counts are recorded as experiment data. Furthermore, in our experiment we need to subtract about $\frac{2}{3}$ coincidence counts as backgrounds, which makes data collection time even longer. However, if we consider more general case of many photonic qubits and spontaneous parametric down-conversion sources being replaced by single photon sources in the future, as shown in Fig. 1, the number of two-photon interferometers only increases linearly with qubit numbers and data collection time would not increase exponentially either. On the other hand, the exponentially increasing experi-

ment resources for tomography is inevitable. That makes our method more suitable for multi-photon case than quantum state tomography. In this context, it is meaningful for us to give a proof-in-principle experimental demonstration.

In summary, we for the first time experimentally measured the lower and upper bounds of concurrence for a set of two-qubit mixed states using twofold copy parity projection measurement method. The measured results are compatible with the results evaluated from conventional quantum state tomography. The technique we used to provide two faithful copies of an unknown mixed states is perfectly ignorant of the specific mixed state ρ and can be easily generalized for many photonic qubits case in the future. This might be helpful for research on quantum entanglement property of multi-qubit systems.

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